

解析学II (担当:近藤) #2
2007年10月18日

[I] 次の関数の1階偏導関数を全て求めよ.

(1) $x^3 + y^2 + 2$ (2) $e^{xy}(x \cos y - y \sin y)$ (3) $x^2 \cos y - y^2 \cos x$ (4) $\cos(x^2 + xy)$ (5) $\tan^{-1} \frac{x^2}{y}$

[II] 次の関数の1階, 2階偏導関数を全て求めよ.

(1) $\log \sqrt{x^2 + y^2}$ (2) $x \sin xy^2$ (3) $\cos(ax + by)$ (4) $\sin(ax + by)$
(5) $\alpha p^4 + \beta q^3 + \gamma r^2 - 2pq - 2qr - 2rp + 3pqr$ (p, q, r について)

[III] 次の関数の1階, 2階, 3階偏導関数を全て求めよ.

(1) $2x^3 + 5xy + 2y^2$ (2) e^{x^2+xy}

[IV] $w = \frac{a^2\varphi}{\xi a^2 + b^2\varphi}$ であるとき $\frac{\partial w}{\partial a}, \frac{\partial w}{\partial \varphi}, \frac{\partial w}{\partial \xi}, \frac{\partial^2 w}{\partial a \partial \varphi}, \frac{\partial^2 w}{\partial \varphi \partial \xi}$ を全て求めよ.

[V] 次の条件を満たす $g(x), h(x)$ を書け.

(1) $\cos x = 1 + o(g(x)) = 1 + O(h(x)) \quad (x \rightarrow 0)$
(2) $\sqrt{x^2 + x} = o(g(x)) = O(h(x)) \quad (x \rightarrow \infty)$

[VI] 次の評価式の□をうめて評価式を完成せよ.

(1) $e^x = \square + \square x + \square x^2 + O(x^\square) \quad (x \rightarrow 0)$
(2) $\sin x = \square + \square x + \square x^2 + o(x^\square) \quad (x \rightarrow 0)$
(3) $\log(1+x) - \sin x = \square + \square x + \square x^2 + O(x^\square) \quad (x \rightarrow 0)$
(4) $\frac{\sinh x}{x} = \square + \square x + \square x^2 + o(x^\square) \quad (x \rightarrow 0)$
(5) $\sinh x = \frac{1}{2}(e^x - e^{-x}) = \square + \square x + \square x^2 + \square x^3 + O(x^\square) \quad (x \rightarrow 0)$