線形代数学 II（近藤）　演習問題#2

問 1　ベクトルの組 $\boldsymbol{a}_1, \boldsymbol{a}_2, \cdots, \boldsymbol{a}_n$ が一次独立であるか一次従属であるか述べよ.

(1)　$\mathbb{R}^2 \ni \boldsymbol{a}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$

(2)　$\mathbb{R}^2 \ni \boldsymbol{a}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad \boldsymbol{a}_2 = \begin{bmatrix} -1 \\ 3 \end{bmatrix}.$

(3)　$\mathbb{R}^2 \ni \boldsymbol{a}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad \boldsymbol{a}_2 = \begin{bmatrix} -2 \\ -4 \end{bmatrix}.$

(4)　$\mathbb{R}^2 \ni \boldsymbol{a}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad \boldsymbol{a}_2 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \quad \boldsymbol{a}_3 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}.$

(5)　$\mathbb{R}^3 \ni \boldsymbol{a}_1 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}.$

(6)　$\mathbb{R}^3 \ni \boldsymbol{a}_1 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \quad \boldsymbol{a}_2 = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}.$

(7)　$\mathbb{R}^3 \ni \boldsymbol{a}_1 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \quad \boldsymbol{a}_2 = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}, \quad \boldsymbol{a}_3 = \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix}.$

(8)　$\mathbb{R}^3 \ni \boldsymbol{a}_1 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \quad \boldsymbol{a}_2 = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}, \quad \boldsymbol{a}_3 = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}.$

(9)　$\mathbb{R}^3 \ni \boldsymbol{a}_1 = \begin{bmatrix} 0 \\ 2 \\ 5 \end{bmatrix}, \quad \boldsymbol{a}_2 = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}, \quad \boldsymbol{a}_3 = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \quad \boldsymbol{a}_4 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}.$

(10)　$\mathbb{R}^4 \ni \boldsymbol{a}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 2 \end{bmatrix}, \quad \boldsymbol{a}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}.$

(11)　$\mathbb{R}^4 \ni \boldsymbol{a}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 2 \end{bmatrix}, \quad \boldsymbol{a}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad \boldsymbol{a}_3 = \begin{bmatrix} 3 \\ 2 \\ 1 \\ -1 \end{bmatrix}, \quad \boldsymbol{a}_4 = \begin{bmatrix} -1 \\ 2 \\ 3 \\ 1 \end{bmatrix}.$

(12)　$\mathbb{R}^4 \ni \boldsymbol{a}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 2 \end{bmatrix}, \quad \boldsymbol{a}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad \boldsymbol{a}_3 = \begin{bmatrix} 3 \\ 2 \\ 1 \\ -1 \end{bmatrix}, \quad \boldsymbol{a}_4 = \begin{bmatrix} -1 \\ 1 \\ -1 \\ 2 \end{bmatrix}.$

問 2　ベクトル $\boldsymbol{u}_1, \boldsymbol{u}_2, \cdots, \boldsymbol{u}_m$ が一次独立のとき，$\boldsymbol{v}_1, \boldsymbol{v}_2, \cdots, \boldsymbol{v}_n$ は一次独立であるか一次従属であるか述べよ.

(1) $\begin{cases} \boldsymbol{v}_1 = \boldsymbol{u}_1 - \boldsymbol{u}_2 \\ \boldsymbol{v}_2 = \boldsymbol{u}_1 + \boldsymbol{u}_2 \end{cases}$

(2) $\begin{cases} \boldsymbol{v}_1 = \boldsymbol{u}_1 - 2\boldsymbol{u}_2 \\ \boldsymbol{v}_2 = -3\boldsymbol{u}_1 + 6\boldsymbol{u}_2 \end{cases}$

(3) $\begin{cases} \boldsymbol{v}_1 = \boldsymbol{u}_1 - \boldsymbol{u}_2 \\ \boldsymbol{v}_2 = \boldsymbol{u}_1 + \boldsymbol{u}_2 \\ \boldsymbol{v}_3 = 2\boldsymbol{u}_1 + 3\boldsymbol{u}_2 \end{cases}$

(4) $\begin{cases} \boldsymbol{v}_1 = \boldsymbol{u}_1 - \boldsymbol{u}_2 + \boldsymbol{u}_3 \\ \boldsymbol{v}_2 = \boldsymbol{u}_1 + \boldsymbol{u}_2 - \boldsymbol{u}_3 \end{cases}$

(5) $\begin{cases} \boldsymbol{v}_1 = \boldsymbol{u}_1 - \boldsymbol{u}_2 + \boldsymbol{u}_3 \\ \boldsymbol{v}_2 = -\boldsymbol{u}_1 + \boldsymbol{u}_2 - \boldsymbol{u}_3 \end{cases}$

(6) $\begin{cases} \boldsymbol{v}_1 = \boldsymbol{u}_1 - \boldsymbol{u}_2 + \boldsymbol{u}_3 \\ \boldsymbol{v}_2 = \boldsymbol{u}_1 + 3\boldsymbol{u}_2 - \boldsymbol{u}_3 \\ \boldsymbol{v}_3 = 2\boldsymbol{u}_1 - \boldsymbol{u}_2 + 2\boldsymbol{u}_3 \end{cases}$

(7) $\begin{cases} \boldsymbol{v}_1 = \boldsymbol{u}_1 - \boldsymbol{u}_2 + \boldsymbol{u}_3 \\ \boldsymbol{v}_2 = \boldsymbol{u}_1 + 3\boldsymbol{u}_2 - \boldsymbol{u}_3 \\ \boldsymbol{v}_3 = 3\boldsymbol{u}_1 + \boldsymbol{u}_2 + \boldsymbol{u}_3 \end{cases}$

(8) $\begin{cases} \boldsymbol{v}_1 = \boldsymbol{u}_1 - \boldsymbol{u}_2 + \boldsymbol{u}_3 \\ \boldsymbol{v}_2 = \boldsymbol{u}_1 + 3\boldsymbol{u}_2 - \boldsymbol{u}_3 \\ \boldsymbol{v}_3 = 2\boldsymbol{u}_1 - \boldsymbol{u}_2 + 2\boldsymbol{u}_3 \\ \boldsymbol{v}_4 = \boldsymbol{u}_1 + 2\boldsymbol{u}_2 + \boldsymbol{u}_3 \end{cases}$

(9) $\begin{cases} \boldsymbol{v}_1 = \boldsymbol{u}_1 - \boldsymbol{u}_2 + \boldsymbol{u}_3 + \boldsymbol{u}_4 \\ \boldsymbol{v}_2 = \boldsymbol{u}_1 + 3\boldsymbol{u}_2 - \boldsymbol{u}_3 - 2\boldsymbol{u}_4 \\ \boldsymbol{v}_3 = 2\boldsymbol{u}_1 - \boldsymbol{u}_2 + 2\boldsymbol{u}_3 - \boldsymbol{u}_4 \end{cases}$

(10) $\begin{cases} \boldsymbol{v}_1 = \boldsymbol{u}_1 - \boldsymbol{u}_2 + \boldsymbol{u}_3 + \boldsymbol{u}_4 \\ \boldsymbol{v}_2 = \boldsymbol{u}_1 + \boldsymbol{u}_2 - \boldsymbol{u}_3 - \boldsymbol{u}_4 \\ \boldsymbol{v}_3 = 3\boldsymbol{u}_1 - \boldsymbol{u}_2 + \boldsymbol{u}_3 + \boldsymbol{u}_4 \end{cases}$

(11) $\begin{cases} \boldsymbol{v}_1 = \boldsymbol{u}_1 - \boldsymbol{u}_2 + \boldsymbol{u}_3 + \boldsymbol{u}_4 \\ \boldsymbol{v}_2 = -\boldsymbol{u}_1 + 2\boldsymbol{u}_2 - \boldsymbol{u}_3 - \boldsymbol{u}_4 \\ \boldsymbol{v}_3 = 2\boldsymbol{u}_1 - \boldsymbol{u}_2 - 2\boldsymbol{u}_3 + 2\boldsymbol{u}_4 \\ \boldsymbol{v}_4 = \boldsymbol{u}_1 + 3\boldsymbol{u}_2 + \boldsymbol{u}_3 + 3\boldsymbol{u}_4 \end{cases}$

(12) $\begin{cases} \boldsymbol{v}_1 = \boldsymbol{u}_1 - \boldsymbol{u}_2 + \boldsymbol{u}_3 + \boldsymbol{u}_4 \\ \boldsymbol{v}_2 = \boldsymbol{u}_1 + 3\boldsymbol{u}_2 + 2\boldsymbol{u}_3 - 2\boldsymbol{u}_4 \\ \boldsymbol{v}_3 = 2\boldsymbol{u}_1 + \boldsymbol{u}_2 - \boldsymbol{u}_3 + \boldsymbol{u}_4 \\ \boldsymbol{v}_4 = 5\boldsymbol{u}_1 + 2\boldsymbol{u}_2 + 3\boldsymbol{u}_3 + \boldsymbol{u}_4 \end{cases}$