

問 1 次の基底を正規直交化せよ .

$$(1) \mathbb{R}^2 \ni \mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 0 \\ -2 \end{bmatrix} .$$

$$(2) \mathbb{R}^2 \ni \mathbf{v}_1 = \begin{bmatrix} 5 \\ -3 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -4 \\ 3 \end{bmatrix} .$$

$$(3) \mathbb{C}^2 \ni \mathbf{v}_1 = \begin{bmatrix} 1+i \\ 3 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -3i \\ 5+4i \end{bmatrix} .$$

$$(4) \mathbb{R}^3 \ni \mathbf{v}_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 5 \\ -2 \\ 2 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 3 \\ -4 \\ 3 \end{bmatrix} .$$

$$(5) \mathbb{R}^3 \ni \mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} .$$

$$(6) \mathbb{R}^3 \ni \mathbf{v}_1 = \begin{bmatrix} 3 \\ 5 \\ 0 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} .$$

$$(7) \mathbb{R}^3 \ni \mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 5 \\ 3 \\ -1 \end{bmatrix} .$$

$$(8) \mathbb{C}^3 \ni \mathbf{v}_1 = \begin{bmatrix} i \\ 0 \\ 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 5+4i \\ -2+3i \\ 1-2i \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 4 \\ -2i \\ 0 \end{bmatrix} .$$

$$(9) \mathbb{R}^4 \ni \mathbf{v}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 2 \\ 3 \\ 1 \\ 0 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} -1 \\ -2 \\ 1 \\ 2 \end{bmatrix}, \mathbf{v}_4 = \begin{bmatrix} -1 \\ 0 \\ 1 \\ -2 \end{bmatrix} .$$

$$(10) \mathbb{R}^4 \ni \mathbf{v}_1 = \begin{bmatrix} 2 \\ -2 \\ 0 \\ 0 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 3 \\ 0 \\ 1 \\ -1 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} -2 \\ 4 \\ 2 \\ 5 \end{bmatrix}, \mathbf{v}_4 = \begin{bmatrix} 0 \\ 5 \\ 3 \\ -1 \end{bmatrix} .$$

$$(11) \mathbb{C}^4 \ni \mathbf{v}_1 = \begin{bmatrix} 2i \\ 0 \\ 0 \\ 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 0 \\ 4i \\ 2 \\ -1+3i \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 2 \\ -i \\ 2+i \\ 3 \end{bmatrix}, \mathbf{v}_4 = \begin{bmatrix} i \\ 0 \\ 2 \\ 5i \end{bmatrix} .$$